Problem 83

A marathon runner completes a 42.188-km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the percent uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

Solution

Part (a)

Use the percent uncertainty formula.

$$\text{Percent Uncertainty} = \frac{\delta A}{A} \times 100\%$$

Plug in the given numbers for distance.

$$\text{Percent Uncertainty} = \frac{25\text{ m}}{42.188\text{ km} \times \frac{1000\text{ m}}{1\text{ km}}} \times 100\% \approx 0.059\%$$

Part (b)

Use the percent uncertainty formula.

$$\text{Percent Uncertainty} = \frac{\delta A}{A} \times 100\%$$

Plug in the given numbers for time.

$$\text{Percent Uncertainty} = \frac{1\text{ s}}{12\text{ s} + 30\text{ min} \times \frac{60\text{ s}}{1\text{ min}} + 2\text{ hr} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{60\text{ s}}{1\text{ min}}} \times 100\% \approx 0.01\%$$

Part (c)

Calculate the maximum speed by using the upper bound for distance and the lower bound for time.

$$\text{Maximum Speed:} \quad \frac{42.188\text{ km} \times \frac{1000\text{ m}}{1\text{ km}} + 25\text{ m}}{12\text{ s} + 30\text{ min} \times \frac{60\text{ s}}{1\text{ min}} + 2\text{ hr} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{60\text{ s}}{1\text{ min}} - 1\text{ s}} = \frac{42\,213\text{ m}}{9011\text{ s}} \approx 4.685\ \frac{\text{m}}{\text{s}}$$

Calculate the minimum speed by using the lower bound for distance and the upper bound for time.

$$\text{Minimum Speed:} \quad \frac{42.188\text{ km} \times \frac{1000\text{ m}}{1\text{ km}} - 25\text{ m}}{12\text{ s} + 30\text{ min} \times \frac{60\text{ s}}{1\text{ min}} + 2\text{ hr} \times \frac{60\text{ min}}{1\text{ hr}} \times \frac{60\text{ s}}{1\text{ min}} + 1\text{ s}} = \frac{42\,163\text{ m}}{9013\text{ s}} \approx 4.678\ \frac{\text{m}}{\text{s}}$$

The average speed is therefore roughly

$$\frac{4.685\ \frac{\text{m}}{\text{s}} + 4.678\ \frac{\text{m}}{\text{s}}}{2} \approx 4.681\ \frac{\text{m}}{\text{s}}.$$

Part (d)

Its uncertainty is roughly

$$\frac{4.685\ \frac{\text{m}}{\text{s}} - 4.678\ \frac{\text{m}}{\text{s}}}{2} \approx 0.003\ \frac{\text{m}}{\text{s}}.$$